Grzegorz Zadora\textsuperscript{1)}, Daniel Ramos\textsuperscript{2)}

*Evaluation of evidence value of refractive index – influence of selection of proper database*

\textsuperscript{1)} Institute of Forensic Research, Krakow, Poland

\textsuperscript{2)} ATVS – Biometric Recognition Group, Escuela Politecnica Superior, Universidad Autonoma de Madrid, Spain
Glass analysis for forensic purposes

<table>
<thead>
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* = <2 Sigma

<0.2mm
Glass analysis for forensic purposes

\[ \Delta RI = (RI_a - RI_b) \]

- \( RI_b \) - refractive index measured before annealing process
- \( RI_a \) - refractive index measured after annealing process

<0.2mm
Glass analysis for forensic purposes - comparison problem

Could they have come from the control sample?

Recovered sample

Control sample
Likelihood ratio

**Important factors**

**Similarity** of compared objects (evidence and control material).

**Information about the rarity** of physico-chemical properties determined for compared samples in the relevant population.

**Sources of analytical errors:** within and between source variability.

\[
LR = \frac{Pr(E|\theta_p)}{Pr(E|\theta_d)}
\]

- **\( LR > 1 \)** - support the prosecutor hypothesis \( H_p \)
  \[ Pr(E|\theta_p) > Pr(E|\theta_d) \]

- **\( LR = 1 \)** - support neither
  \[ Pr(E|\theta_p) = Pr(E|\theta_d) \]

- **\( LR < 1 \)** - support the defence hypothesis \( H_d \)
  \[ Pr(E|\theta_p) < Pr(E|\theta_d) \]

The larger (the lower) the LR value, the stronger the support for \( \theta_p \) (\( \theta_d \)).
LR – two levels of variation

Univariate data (RI) - normal distribution

\[ LR \approx \frac{m^{\frac{1}{2}} \tau}{2^{\frac{1}{2}} \sigma} \exp \left\{ - \frac{m(x - \bar{y})^2}{4\sigma^2} \right\} \exp \left\{ \frac{(z - \mu)^2}{2\tau^2} \right\} \]

LR NOR

Univariate data (RI) - kernel density estimation

\[ LR = \frac{K \exp \left\{ - \frac{(x - \bar{y})^2}{2a^2\sigma^2} \right\} \sum_{i=1}^{k} \exp \left\{ - \frac{(m + n)(w - r_i)^2}{2\sigma^2 + (m + n)s^2\lambda^2} \right\}}{\sum_{i=1}^{k} \exp \left\{ - \frac{m(x - r_i)^2}{2(\sigma^2 + ms^2\lambda^2)} \right\} \sum_{i=1}^{k} \exp \left\{ - \frac{n(y - r_i)^2}{2(\sigma^2 + ns^2\lambda^2)} \right\}} \]

LR KDE
Evidence evaluation using LR values is related with the use of population databases as evaluation of evidence value requires process of the assessment of the rarity of the evidence and this information is also used in the aim of evaluation of between-object variability.

Therefore, the selection of a proper database (relevant population) is one of the crucial points during evaluation of evidence value of physicochemical data.
Glass analysis for forensic purposes - experiments

Samples:
- Experiment ID: pl pl
- Experiment ID: pl uk
- Experiment ID: ww
- Experiment ID: wp

Background:
- Polish flag
- British flag
Glass analysis for forensic purposes - experiments

Samples:
- Experiment ID: pw
- Experiment ID: pp
- Experiment ID: ww
- Experiment ID: wp
Glass analysis for forensic purposes - experiments

Same-source experiments

Fragments from sample A

Recovered sample results from 2 fragments

Control sample results from 2 fragments

Expected: Support to the correct hypothesis: $LR > 1$

Misleading evidence: Support to the wrong hypothesis: $LR < 1$
Glass analysis for forensic purposes - experiments

Different-source experiments

Fragments from sample A

Control sample
All 4 fragments

Fragments from sample B

Recovered sample
All 4 fragments

Expected: Support to the correct hypothesis: $LR < 1$

Misleading evidence: Support to the wrong hypothesis: $LR > 1$
Glass analysis for forensic purposes

LR distributions and rates of misleading evidence

![Graph showing LR distributions and rates of misleading evidence](image)
Glass analysis for forensic purposes - accuracy

- The LR has a meaning by itself
  - *Degree of support* to the previous opinion
  - LR is the weight of the evidence $E$

\[
\frac{Pr(\theta_p)}{Pr(\theta_d)} \cdot \frac{Pr(E|\theta_p)}{Pr(E|\theta_d)} = \frac{Pr(\theta_p | E)}{Pr(\theta_d | E)}
\]

- Inferred posterior probabilities must be **accurate**
- But what’s **accuracy**?
Empirical Cross-Entropy (ECE): evidence evaluation performance

\(ECE\) is the prior-weighted average value of a strictly proper scoring rule

Empirical approach: experimental test

- Generate **same-source** comparisons (\(\theta_p\) is true)
- Generate **different-source** comparisons (\(\theta_d\) is true)

\[
ECE = -P(\theta_p) \frac{1}{N_p} \sum_{j \in \text{same-source}} \log_2 P(\theta_p | e_j) - P(\theta_d) \frac{1}{N_d} \sum_{j \in \text{diff-source}} \log_2 P(\theta_d | e_j)
\]

1) It depends on the prior. The forensic scientist cannot compute its value.
2) Solution: the \(ECE\) plot. Prior-dependent representation.


Empirical Cross-Entropy (ECE): evidence evaluation performance

**ECE curve (solid):** overall performance

The higher its value, the worse the method.

**Calibrated (dashed):** discriminating power

Difference among ECE and Calibrated is the *calibration* performance.

**Neutral, always LR=1 (dotted):** a method that does not take into account the evidence.
Glass analysis for forensic purposes - results

**Background:**

Variable: R Ib

**False**

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Glass analysis for forensic purposes - results

Background:

Variable: R1b

Background:

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Samples: 2.7% 0.0%
Samples: 3.6% 1.2%

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Samples: 2.2% 1.1%
Samples: 3.0% 2.4%

LR KDE
Glass analysis for forensic purposes - results

Background:

Samples:

Samples:

LR KDE

LR NOR
Glass analysis for forensic purposes - results

**Background:**

**Samples:**

**Samples:**

**LR KDE**

**LR NOR**
Glass analysis for forensic purposes - conclusion

1) RIb variable seems quite robust to the variation among Polish and British float glass databases.
Glass analysis for forensic purposes - results

**Background:**

**Variable: R1b**

**False**

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Glass analysis for forensic purposes - results

Samples: \[
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\end{align*}
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Background:

\[
\begin{align*}
\text{RI}_b & \quad \text{RI}_b & \\
\end{align*}
\]
Glass analysis for forensic purposes - conclusion

1) R1b variable seems quite robust to the variation among Polish and British float glass databases.

2) R1b variable seems quite robust to the variation among containers and float glass databases.
Glass analysis for forensic purposes - results

**Background:**

Variable: $\Delta R I$

**False**

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**LR KDE**
Glass analysis for forensic purposes - results

Samples: ![Glass Bottles]  ΔRI  Samples: ![Glass Bottles]

Background: ![Glass Bottles]

Samples: ![Glass Bottles]  ΔRI  Samples: ![Glass Bottles]

Background: ![Glass Bottles]
1) RIB variable seems quite robust to the variation among Polish and British float glass databases.

2) RIB variable seems quite robust to the variation among containers and float glass databases.

3) dRI variable is very sensitive to the type of glass object analysed, which forces careful selection of the background database when dealing with such variable in comparison tasks.
Acknowledgments

The authors wish to thank Jim Haworth, Key Forensic Services, University of Warwick Science Park, Coventry, UK, for delivery of samples of British float glass.
Grzegorz Zadora¹), Daniel Ramos²)

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Neutral, always LR=1 (doted): a method that does not take into account the evidence

**Separation of roles:**
- **Forensic scientist:** ECE computation for a wide range of priors
  Because the scientist cannot set the prior…
- **Fact finder:** prior establishment and measure of ECE in the plot