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Kinematical Analysis of Synthetic Dynamic Signatures Using the Sigma-Lognormal Model

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***Abstract*—The kinematical information present in synthetically generated signatures is analyzed using the Sigma-Lognormal model and compared to the kinematical properties of real samples. Experiments are carried out on totally independent development and test sets and show a high degree of similarity between humanly produced and artificial signatures. One particular flaw is found in the velocity profile of synthetic signatures. Two possible solutions are proposed to improve the synthetic generation method using the Kinematic Theory of rapid human movements.**

***Keywords*-signature; synthetic generation; kinematical information; Sigma-Lognormal model;**

I. Introduction

With the increasing importance that biometric security systems are acquiring in today's society and their introduction in many daily applications, a growing interest is arising for the generation of synthetic biometric traits such as voice [1], fingerprints [2], iris [3], handwriting [4], or signature [5]. In many cases, these synthetically generated traits have proven to present, when used in automatic recognition systems, a very similar performance to that of the real ones [6]. In addition, synthetic databases have the clear advantage over real datasets of presenting a nearly effort-free generation process in comparison to the time-consuming and complicated process of real acquisition campaigns. All these characteristics make synthetic samples very useful for the performance evaluation of biometric systems.

However, in spite of their advantages in many potential applications, the generation of realistic synthetic biometric data still represents a very complex problem: modeling the information contained in a certain biometric trait as well as the inter-class and intra-class variations found in real databases (i.e., variation between samples of different subjects, and variation between samples of the same subject, respectively). Furthermore, one of the problems to be faced when generating synthetic biometric traits is the definition of objective ways to quantify the *realism* of the produced artificial samples. That is, to define the set of needs that a synthetic sample has to satisfy in order to be recognized and treated by automatic verification systems as a physically collected trait.

For the particular case of on-line signature, we can distinguish three different requirements that should be met by synthetic samples: i) synthetic signatures should look as close as possible to real signatures, ii) synthetic signatures should present the same information as real signatures, and iii) synthetic signature databases should present the same inter- and intra-user variability as real signature datasets.

This work addresses the second of these requirements focusing, in particular, on the analysis of the kinematic information contained on real and synthetic signatures with two objectives:

- **Objective 1**. Determine to what extent this type of information is present in a similar manner in both types of samples (real and synthetic).
- **Objective 2**. Use the knowledge acquired from the experiments carried out to complete the previous objective, to propose a way to improve the generation method of synthetic dynamic signatures.

For this purpose, we will use the Kinematic Theory of rapid human movements, which was initially proposed for the analysis of handwriting [7], [8] and then used for other applications [9], [10]. This theory models in a realistic way the different neuromuscular processes involved in the production of handwriting through the application of the Sigma-Lognormal model which can be used to parameterize each of the strokes involved in the signing process [9].

The theory will be applied to analyze the kinematic properties of real signatures from the BiosecurID database [11], and synthetic signatures generated according to the algorithm proposed in [5]. Although different methods have been proposed in the literature for the generation of artificial on-line signatures, the great majority follows *duplicated samples* strategies which are able to produce different synthetic impressions of a single *real* signature [12], [13], [14], but cannot generate totally *synthetic individuals*. In a previous work [5], a fully automatic model-based method was proposed for the generation of totally synthetic datasets (i.e., no real samples are used in the process); moreover,

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quantitative results were reported about the suitability of the methodology for synthetic signature generation.

The paper is structured as follows. The Sigma-Lognormal model is reviewed in Sect. II, while the method for synthetic signature generation is revised in Sect. III. The experimental protocol is presented in Sect. IV, and results are given in Sect. V. Conclusions are finally drawn in Sect. VI.

II. The Sigma-Lognormal model

The Kinematic Theory of rapid human movements, which was first introduced in [7], [8], relies on the Sigma-Lognormal model to represent the information of both the motor commands and the timing properties of the neuromuscular system involved in the production of complex movements like signatures.

The Sigma-Lognormal model considers the resulting speed of a single stroke j as having a lognormal shape Λ scaled by a command parameter (D) and time-shifted by the time occurrence of the command (t_0).

$$
\begin{aligned}
|\vec{v}_j(t;P_j)| &= D_j\Lambda(t-t_{0j};\mu_j,\sigma_j^2) = \\
&= \tfrac{D_j}{\sigma(t-t_{0j})\sqrt{2\pi}}\exp\{\tfrac{[\ln(t-t_{0j})-\mu_j]^2}{-2\sigma_j^2}\},
\end{aligned}
$$

where $P_j = [D_j, t_{0j}, \mu_j, \sigma_j, \theta_{sj}, \theta_{ej}]$ represents the set of Sigma-Lognormal parameters:

- D_j: the amplitude of the input commands.
- t_{0j}: the time occurrence of the input commands, a time-shift parameter.
- μ_j: the log-time delays, the time delays of the neuromuscular system expressed on a logarithmic time scale.
- σ_j: the log-response times, which are the response times of the neuromuscular system expressed on a logarithmic time scale.
- θ_{sj}: starting angles of the circular trajectories described by the lognormal model along a pivot.
- θ_{ej}: ending angles of the circular trajectories described by the lognormal model along a pivot.

Additionally, from the hypothesis that every lognormal stroke represents the movement as happening along a pivot, the angular position can be computed as

$$
\phi_j(t;P_j) = \theta_{sj} + \frac{\theta_{ej}-\theta_{sj}}{D_j}\int_0^t |\vec{v}_j(\tau;P_j)|d\tau,
$$

In this context, a signature can be seen as the output of a generator that produces a set of individual strokes superimposed in time. The resulting complex trajectory can be modeled as a vectorial summation of lognormals (being N_{LN} the total number of lognormal curves in which the signature is decomposed):

$$
\vec{v}(t) = \Sigma\Lambda(t) = \sum_{j=1}^{N_{LN}} \vec{v}_j(t;P_j).
$$

The velocity components in the Cartesian space can be calculated from the tangential speed as:

$$
\begin{aligned}
\vec{v_x}(t) &= \sum_{j=1}^{N_{LN}} |\vec{v}_j(t;P_j)|\cos(\phi_j(t;P_j)),\\
\vec{v_y}(t) &= \sum_{j=1}^{N_{LN}} |\vec{v}_j(t;P_j)|\sin(\phi_j(t;P_j)).
\end{aligned}
$$

The reconstruction error of a velocity profile using the Sigma-Lognormal parameters can be evaluated by computing the SNR between the reconstructed specimen and the original one [15]:

$$
10\log(\frac{\int_{t_s}^{t_e}[v_{xo}^2(t)+v_{yo}^2(t)]dt}{\int_{t_s}^{t_e}[(v_{xo}(t)-v_{xa}(t))^2+(v_{yo}(t)-v_{ya}(t))^2]dt}), \quad (1)
$$

where t_s and t_e are respectively the starting and ending times of the signature, and the subindex o refers to the original velocity profile (x or y) while a corresponds to the artificially reconstructed functions.

This fitness evaluation metric will be used in the experiments (Sect. V-B) to estimate how well the velocity function of synthetic signatures is reconstructed following the Sigma-Lognormal model in comparison to real samples.

III. Generating dynamic synthetic signatures

In the present contribution we will consider that online handwritten signatures are described by three time sequences, namely: i) the two trajectory functions x and y defining respectively the horizontal and vertical movement of the signing process, and ii) the function p that represents the pressure exerted by the signer at each sampled point.

The synthetic signatures used in the experiments are produced following an algorithm based on a generative model obtained from the spectral analysis of real signatures, and described in [5]. This method follows three steps in order to generate realistic signatures starting from filtered white noise:

- **Step 1**. A parametrical model in the frequency domain is used to colour the white noise and create the synthetic Discrete Fourier Transform (DFT) of the trajectory signals x and y. The parameters that define the model are: i) time duration, ii) number of low-frequency high-energy coefficients (i.e., number of coefficients whose energy exceeds a given threshold), iii) magnitude of these relevant coefficients, iv) magnitude of the remaining DFT coefficients (high-frequency and low-energy). All these parameters are estimated from the BiosecurID database [11], comprising 6,400 real signatures from 400 users collected over 4 acquisition sessions.
- **Step 2**. The Inverse Discrete Fourier Transform (IDFT) is computed and the resulting trajectory signals are processed in the time domain in order to give the

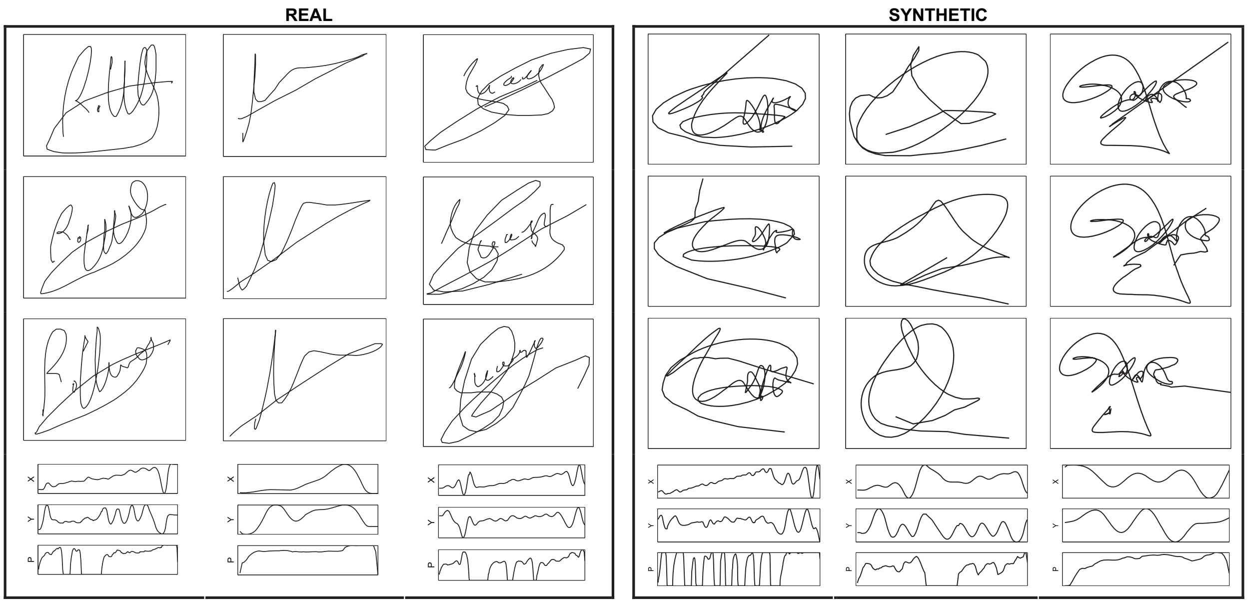

Figure 1. Examples of real and synthetic signatures. The trajectories $x[n]$, $y[n]$, and pressure $p[n]$ sequences correspond to the sample in the first row.

synthetic signatures a more realistic appearance. This processing stage consists of: i) smoothing of the signals, ii) giving the x signal an increasing tendency (as it is the case in most left to right written signatures), iii) adding an artificial round-like flourish at the end of some signatures, iv) translation, rotation and scaling transformations.

- **Step 3**. The pressure function of the signature is generated following the BiosecurID [11] penups distribution and according to the coordinate signals previously created. The penups of the signal are located close to maxima/minima of the y function (as it happens in most of the cases in real signatures) and undesired effects are suppressed (e.g., too long or too short penups, penups at the start or the end of the signature, etc.) The pressure signals are finally quantized to 1024 levels.

Once the three dynamic sequences (x, y, and p) have been created, different samples of that master-signature are generated modeling the user variability for intrasession and intersession samples. The process for generating multiple samples includes: i) scaling the three functions, ii) expanding or contracting its length, and iii) adding smoothed white noise to the trajectory sequences.

IV. Experimental protocol

Three different datasets were used in the experiments. One development set (comprising real signatures) for the estimation of the parameters that define the synthetic generation method, and two test sets one real and one synthetic, to compare the kinematic information present in both of them.

- **Development set**: For the estimation of the parameters which define the synthetic generation method we used the signature data in the BiosecurID multimodal database [11]. This signature corpus includes for each of the 400 users, 16 original samples captured in four acquisition sessions over a six month time span which makes it a very efficient tool to estimate the inter and intrasession variability.
- **Real test set**: As real test set, the dynamic signature data of the MCYT database was used [16]. The signature dataset used in the experiments is formed by 25 original samples for each of the 330 users present in MCYT.
- **Synthetic test set**: The synthetic database produced for the experiments (SDB) was generated following the MCYT structure, comprising 330 different signers with 25 samples per user. The first 5 of those 25 signatures were generated using the intrasession values of the model parameters (estimated on the development set BiosecurID), and the rest with the intersession values.

In Fig. 1 three samples of three real (left) and synthetic (right) signers are shown. Real and synthetic signatures have been extracted from MCYT and SDB respectively.

V. Results

Two different experiments were carried out in order to evaluate to what extent the kinematic information of real signatures is present in a similar manner in synthetic samples. Results from each of the experiments are described in the next sections.

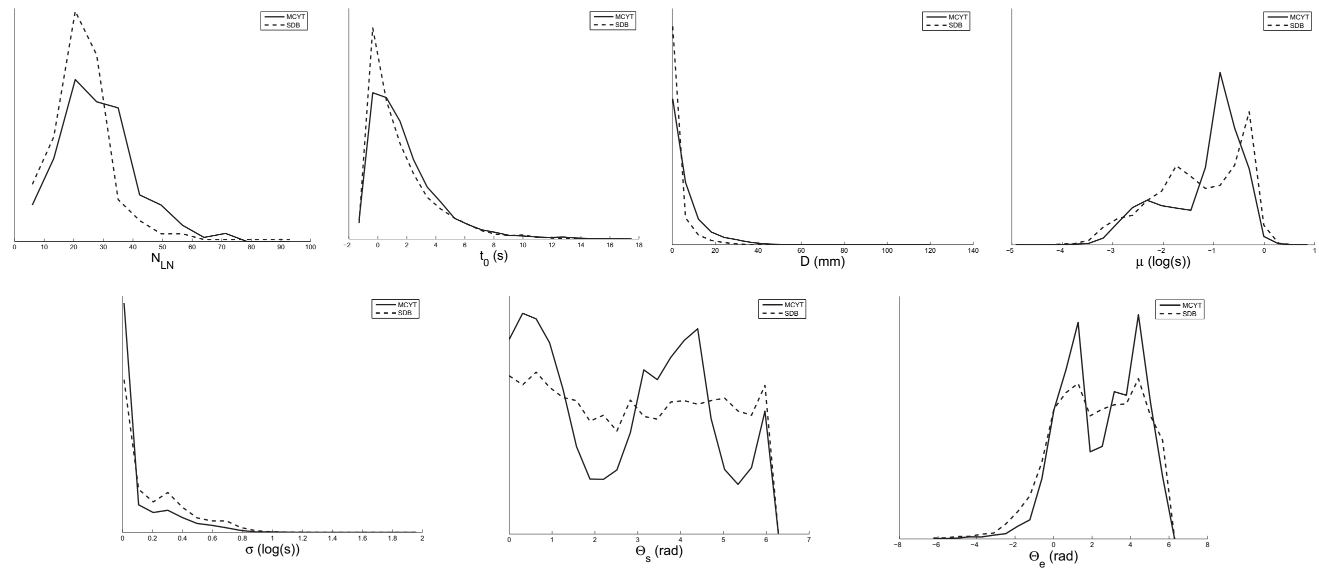

Figure 2. Distributions of the Sigma-Lognormal parameters for synthetic (SDB) and real (MCYT) signatures.

	Percentage of shared kinematic information							
	N_{LN}	t_0	D	μ	σ	θ_s	θ_e	Mean
MCYT–SDB	76.51	90.97	73.36	74.47	78.32	82.81	88.92	80.77

Table I
PERCENTAGE OF SHARED KINEMATIC INFORMATION BETWEEN REAL AND SYNTHETIC DATABASES ACCORDING TO THE SIGMA-LOGNORMAL MODEL.

A. Experiment 1: Sigma-Lognormal parameters comparison

In this first experiment we analyzed the percentage of shared information present in the real and synthetic test sets (SDB and MCYT) according to the set of Sigma-Lognormal features described in Sect. II. With this objective the Sigma-Lognormal parameters were extracted from each signature (six parameters per stroke) in MCYT and from SDB following the method described in [15]. The set of features was completed with a seventh parameter N_{LN} defining the number of lognormal strokes forming a given signature.

In order to give a measure of the common information between MCYT and SDB, the individual distributions of each parameter for real and synthetic samples were computed and the amount of information shared by both type of signatures (real and synthetic) was estimated as the area common to both distributions. That is, given the real and synthetic distributions R_i and S_i with $i = 1 \dots 7$, corresponding to each of the 6 lognormal features plus L_{LN}, the shared information for one particular parameter is defined as $\mathrm{A}_i = 1 - 1/2 \int |R_i - S_i|$, while the total amount of common information for real and synthetic signatures is computed as $\mathrm{A} = 1/7 \sum_i \mathrm{A}_i$. These amounts of shared information between real and synthetic signatures are given in Table I.

In order to supply also with a visual comparison between distributions in addition to the quantitative measures, the real (solid) and synthetic (dashed) individual distributions for each of the Sigma-Lognormal parameters are shown in Fig. 2.

The main differences that can be pointed out from the distributions depicted in Fig. 2 are that synthetic strokes are: i) a little bit shorter (see D distribution), ii) slightly slower (see μ and σ distributions), and iii) they do not present any predominant starting or ending direction while real strokes tend to begin and finish with an angle close to 0 or π (see θ_s and θ_e distributions).

It should also be noticed that synthetic signatures are formed by a slightly fewer number of strokes (see N_{LN} distribution) which combined with the previous observations i) and ii) means that compared to real signatures artificial samples are a little bit shorter from a spatial point of view, but of the same duration.

Apart from the differences highlighted above, the results given in Table I and Fig. 2 show that, from a general point of view, the kinematic information of the synthetic signatures is very similar (over 80%) to that found in real samples.

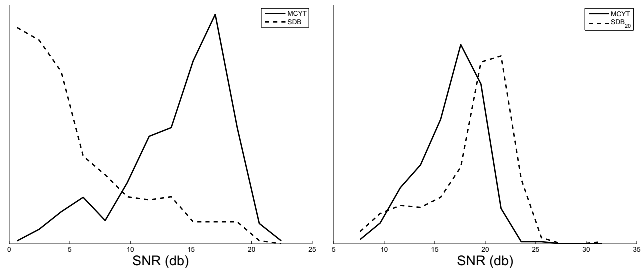

Figure 3. Distributions of the reconstruction error of the velocity profile for real signatures (MCYT) compared to synthetic signatures (SDB), left plot, and to synthetic signatures discarding the initial and final 20 samples (SDB_{20}), right plot.

B. Experiment 2: Reconstruction error

In this second experiment, the velocity profile of real and synthetic signatures was reconstructed from their set of Sigma-Lognormal parameters and the reconstruction error evaluated according to the SNR metric defined in Eq. 1. The resulting distributions of the reconstruction errors committed on real and artificial samples is depicted in Fig. 3 (left), where we can observe that, although the Sigma-Lognormal features (as proved in experiment 1) are very similar in both types of signatures, the reconstruction of the velocity profiles is much worse in the case of the synthetic samples.

This degradation in the reconstruction quality is due to very high velocity peaks that can be observed at the beginning and ending parts of the original velocity profile of many of the synthetic signatures (see solid lines in Fig. 4), which do not correspond to the typical movement of real signers, where the velocity function starts and finishes at zero (or near zero) values.

These abnormal speed artifacts can be corrected by discarding a few initial and final samples of the synthetic signatures. In Fig. 3 (right) we show the SNR distributions for the reconstructed velocity profiles of the complete real signatures in MCYT and the synthetic samples in SDB without considering the first and last 20 samples (SDB_{20}). It can be observed that most of the reconstruction error was concentrated in the erased samples as in this case both distributions are very similar, the error committed in the reconstruction of the synthetic samples being even a little bit lower.

Although the previous approach (discarding the spurious samples) has proven its efficiency, it artificially modifies the synthetic signatures making them slightly shorter. A better way of approaching the problem would be to use the reconstructed Sigma-Lognormal velocity function (from which the coordinate signals x and y may be recovered) as the new master signature for the generation of duplicated samples. In this way, by combining the spectral analysis and the Kinematic Theory of rapid human movements, the synthetic generation method would be improved as the produced signatures would not present these type of high velocity undesired artifacts (see the reconstructed velocity functions in Fig. 4, dashed lines) and at the same time they would encompass not only the topological and geometric information, but also the full kinematic properties of real signatures. This solution is beyond the scope of the present study and will eventually be part of a follow up project.

VI. Conclusions

The kinematic information present in synthetic on-line signatures has been analyzed using the Kinematic Theory of rapid human movements and compared to the dynamic properties of real samples.

The experiments, carried out using totally independent development and test sets, have proven that to a very high extent this type of information is shared in a very similar way by both type of signatures (real and synthetic). However, in spite of the clear similarities observed, a flaw has been detected in the velocity profile of synthetic signatures and two possible solutions have been proposed in order to improve the synthetic generation method used in the experiments.

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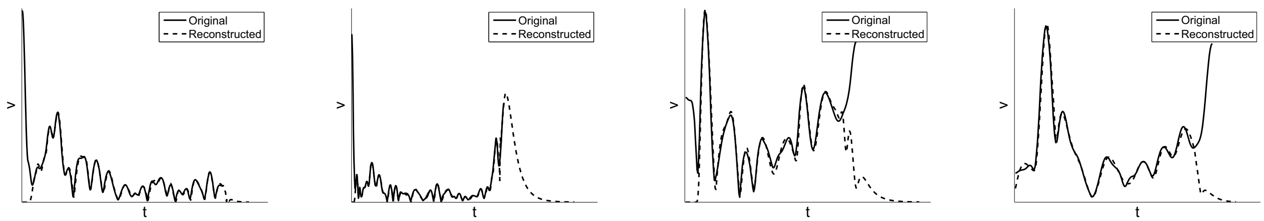

Figure 4. Original and reconstructed velocity functions of 4 example synthetic signatures.

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